Holographic Ricci dark energy: Interacting model and cosmological constraints

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We extend the holographic Ricci dark energy model to include some direct, non-gravitational interaction between dark energy and dark matter. We consider three phenomenological forms for the interaction term Q in the model, namely, Q is taken proportional to the Hubble expansion rate and the energy densities of dark sectors (taken to be $\rho_{\rm de}$, $\rho_{\rm m}$, and $\rho_{\rm de}+\rho_{\rm m}$, respectively). We obtain a uniform analytical solution to the three interacting models. Furthermore, we constrain the models by using the latest observational data, including the 557 Union2 type Ia supernovae data, the cosmic microwave background anisotropy data from the 7-yr WMAP, and the baryon acoustic oscillation data from the SDSS. We show that in the interacting models of the holographic Ricci dark energy, a more reasonable value of $\Omega_{\rm m0}$ will be obtained, and the observations favor a rather strong coupling between dark energy and dark matter.

Our universe is undergoing an accelerated expansion, which was discovered via the observations of distant type Ia supernovae (SN) [1], about a decade ago. Combined analysis of various observational data points to a cosmological model that contains dark energy, an exotic cosmic component with negative pressure. What is more, dark energy is dominating the evolution of the current universe: it occupies about 73% of the total energy of the universe. However, currently, the nature of dark energy is still enigmatic, and so the revelation of the nature of dark energy raises one of the biggest challenges for the modern fundamental science.

The cosmological constant Λ, first introduced by Einstein in 1917 [2], is an important candidate for dark energy, because it can provide a nice explanation for the accelerating universe and can fit the observational data well. Nevertheless, the cosmological constant is suffering from severe theoretical challenges [3]: one cannot explain why the theoretical value of Λ from the current framework of physics is greater than the observational value by many orders of magnitude, and why the densities of dark energy and matter are in the same order just today while they evolve with rather different ways. Besides the cosmological constant, there is a fairly attractive idea that the dark energy may not be a rigescent constant but a dynamically evolving component. In fact, many dynamical models of dark energy have been proposed and studied in detail. Among the many dynamical models of dark energy, the holographic model of dark energy [4–6] is very attractive since it originates from the consideration of the holographic principle [7] of quantum gravity. It is widely believed that the physical nature of dark energy is in deep connection with the underlying quantum gravity theory. Thus, the theoretical and phenomenological studies on holographic dark energy may provide significant clues for the bottom-up exploration of a full quantum theory of gravitation.

When considering gravity in a quantum field system, the conventional local quantum field theory will break down due to the too many degrees of freedom that would cause the formation of a black hole. According to the holographic principle, one may put an energy bound on the vacuum energy density, $\rho_{\rm vac}L^3 \le M_{\rm Pl}^2 L$ [8], where $\rho_{\rm vac}$ is the vacuum energy density and $M_{\rm Pl}$ is the reduced Planck mass. This bound implies that the total energy in a spatial region with size L should not exceed the mass of a black hole with the same size. The largest size compatible with this bound is the infrared

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(IR) cutoff size of this effective field theory. Evidently, the holographic principle gives rise to a dark energy model basing on the effective quantum field theory with a UV/IR duality. From the UV/IR correspondence, the UV problem of dark energy can be converted into an IR problem. A given IR scale can saturate that bound, and so one can write the dark energy density as $\rho_{\rm de} = 3c^2 M_{\rm Pl} L^{-2}$ [4], where c is a phenomenological parameter (dimensionless) characterizing all of the uncertainties of the theory. Note that, hereafter, we use $\rho_{\rm de}$ to denote the dark energy density. Now, the problem becomes how to choose an appropriate IR cutoff for the theory. Li [4] proposed to choose the event horizon of the universe as the IR cutoff of the theory. This choice generates a successful holographic model of dark energy, explaining both the fine-tuning problem and the cosmic coincidence problem at the same time, in some degree. Subsequently, other holographic models of dark energy basing on the different choices of IR cutoff were also proposed. For example, the choice of the conformal age of the universe leads to the agegraphic dark energy model [9], and the choice of the Ricci scale of the universe gives rise to the holographic Ricci dark energy model [10].

The present paper focuses on the holographic Ricci dark energy (hereafter, abbreviated as RDE) model. In the RDE model, the IR length scale L is given by the average radius of the Ricci scalar curvature $|\mathcal{R}|^{-1/2}$, so in this case the density of the holographic dark energy is $\rho_{\rm de} \propto \mathcal{R}$. In a spatially flat universe, the Ricci scalar of the spacetime is given by $\mathcal{R} = -6(\dot{H} + 2H^2)$, where $H = \dot{a}/a$ is the Hubble expansion rate of the universe, and the dot denotes the derivative with respect to the cosmic time t. Therefore, the density of dark energy can be expressed as [10]

$$\rho_{\rm de} = 3\alpha M_{\rm Pl}^2 (\dot{H} + 2H^2), \tag{1}$$

where α is a dimensionless parameter replacing c^2 of the Li model [4]. Note that, throughout the paper, we consider a flat universe owing to the fact that the flatness of the space is an important prediction of the inflationary cosmology and has been confirmed by observations, e.g., the current constraint is $\Omega_{k0} \sim 10^{-3}$ [11]. So, the Friedmann equation is written as $3M_{\rm Pl}^2 H^2 = \rho_{\rm de} + \rho_{\rm m}$, where $\rho_{\rm m}$ denotes the matter density. The RDE model has been studied extensively. In Ref. [12], the holographic meaning of RDE was revealed by investigating the causal connection scale $R_{\rm CC}$. In Ref. [13], the cosmological constraints on the RDE were studied, and the quintom feature was found. In Ref. [14], it was shown that the existence of the cosmic doomsday in the RDE model would ruin the theoretical foundation of the scenario, and a mend from the consideration of extra-dimension effects could erase the big-rip singularity and leads to a de Sitter finale for the holographic cosmos. For other relevant work, see, e.g., [15].

It is natural to consider the possible interaction between dark energy and dark matter in the RDE model. If dark energy interacts with cold dark matter, the continuity equations for them are

$$\dot{\rho}_{\rm de} + 3H(1+w)\rho_{\rm de} = -Q, \tag{2}$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q, \tag{3}$$

where w is the equation of state (EOS) parameter of dark energy, and Q is the interaction term. Note that, although $\rho_{\rm m}$ includes densities of cold dark matter and baryon matter, in this place we use $\rho_{\rm m}$ to approximately describe dark matter density due to the fact that the density of baryon matter is much less than that of dark matter. In addition, owing to the lack of mechanism of microscopic origin of the interaction, one has to assume the forms of Q phenomenologically. For interacting dark energy model, several forms for Q have been put forward [16]. The most widely used form is $Q \propto H\rho$, where ρ denotes the energy density of the dark sectors, and usually it has three choices, namely, $\rho = \rho_{\rm de}$, $\rho = \rho_{\rm m}$, and $\rho = \rho_{\rm de} + \rho_{\rm m}$. Suwa and Nihei [17] considered the case of $Q \propto H\rho_{\rm de}$ for the interacting Ricci dark energy (IRDE) model. And, they placed the cosmological constraints on this model by using the SN, cosmic microwave background (CMB), and baryon acoustic oscillation (BAO) data. However, it should be pointed out that, in the work of

Suwa and Nihei [17], only one special case of interaction is considered. Moreover, the observational data used in the cosmological constraints are outdated today: 307 SN data from Union dataset, and CMB shift parameter from WMAP 5-year observation. In this paper, we will make some improvements. We will consider a more general interaction term, $Q \propto H\rho$, in the IRDE model, and place the cosmological constraints on the model by employing the latest observational data. Specifically, the interaction term is written as $Q = 3bH\rho$, where ρ denotes $\rho_{\rm de}$, $\rho_{\rm m}$, and $\rho_{\rm de} + \rho_{\rm m}$, respectively, and b is a dimensionless coupling constant. According to our convention, $b > 0$ means that the energy transfer is from dark energy to cold dark matter.

Combining with Eqs. (1) and (3), the Friedmann equation can be expressed as

$$\frac{\alpha}{2}\frac{d^2H^2}{dx^2} - C\frac{dH^2}{dx} - DH^2 \;=\; 0 \tag{4}$$

where $C = 1 - 7\alpha/2 - 3\alpha b/2$ and $D = 3 - 6\alpha - 6\alpha b$ for $Q \propto H\rho_{\rm de}$; $C = 1 - 7\alpha/2 + 3\alpha b/2$ and $D = 6\alpha - 6\alpha b + 3b - 3$ for $Q \propto H\rho_{\rm m}$; $C = 1 - 7\alpha/2$ and $D = 3 - 6\alpha - 3b$ for $Q \propto H(\rho_{\rm de} + \rho_{\rm m})$; and $x = \ln a$ with a being the scale factor of the universe. The solution to Eq. (4) is obtained,

$$\frac{H^2}{H_0^2} \;=\; A_+ e^{\sigma_+ x} + A_- e^{\sigma_- x}, \tag{5}$$

where

$$\sigma_\pm \;=\; \frac{C \pm \sqrt{C^2 + 2D\alpha}}{\alpha}, \tag{6}$$

The initial conditions of Eq. (4) are

$$\left.\frac{H^2}{H_0^2}\right|_{x=0} = 1, \tag{7}$$

and

$$\left.\frac{1}{H_0^2}\frac{dH^2}{dx}\right|_{x=0} = \frac{2}{\alpha}\Omega_{\rm de0} - 4, \tag{8}$$

where $\Omega_{\rm de0} = 1 - \Omega_{\rm m0}$. The constants $A_\pm$ are given by

$$A_\pm \;=\; \pm\frac{2\Omega_{\rm de0} - \alpha(\sigma_\mp + 4)}{\alpha(\sigma_+ - \sigma_-)}. \tag{9}$$

In what follows, we will constrain the IRDE model by using the latest observational data. We will use the 557 SN data from the Union2 dataset, the CMB data from the WMAP 7-year observation, and the BAO data from the SDSS. We will obtain the best-fitted parameters and likelihoods by using a Markov Chain Monte Carlo (MCMC) method.

We use the data points of the 557 Union2 SN compiled in Ref. [18]. The theoretical distance modulus is defined as

$$\mu_{\rm th}(z_i) \equiv 5\log_{10} D_L(z_i) + \mu_0, \tag{10}$$

where $z = 1/a - 1$ is the redshift, $\mu_0 \equiv 42.38 - 5\log_{10} h$ with h the Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1}, and the Hubble-free luminosity distance

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{E(z';\boldsymbol{\theta})}, \tag{11}$$

where $E(z) \equiv H(z)/H_0$, and $\boldsymbol{\theta}$ denotes the model parameters. Correspondingly, the χ^2 function for the 557 Union2 SN data is given by

$$\chi^2_{\rm SN}(\boldsymbol{\theta}) = \sum_{i=1}^{557} \frac{\left[\mu_{\rm obs}(z_i) - \mu_{\rm th}(z_i)\right]^2}{\sigma^2(z_i)}, \tag{12}$$

where σ is the corresponding 1σ error of distance modulus for each supernova. The parameter μ_0 is a nuisance parameter and one can expand Eq. (12) as

$$\chi^2_{\rm SN}(\boldsymbol{\theta}) = A(\boldsymbol{\theta}) - 2\mu_0 B(\boldsymbol{\theta}) + \mu_0^2 C, \tag{13}$$

where $A(\boldsymbol{\theta})$, $B(\boldsymbol{\theta})$ and C are defined in Ref. [19]. Evidently, Eq. (13) has a minimum for $\mu_0 = B/C$ at

$$\tilde{\chi}^2_{\rm SN}(\boldsymbol{\theta}) = A(\boldsymbol{\theta}) - \frac{B(\boldsymbol{\theta})^2}{C}. \tag{14}$$

Since $\chi^2_{\rm SN,\,min} = \tilde{\chi}^2_{\rm SN,\,min}$, instead minimizing $\chi^2_{\rm SN}$ we will minimize $\tilde{\chi}^2_{\rm SN}$ which is independent of the nuisance parameter μ_0.

Next, we consider the cosmological observational data from WMAP and SDSS. For the WMAP data, we use the CMB shift parameter R; for the SDSS data, we use the parameter A of the BAO measurement. It is widely believed that both R and A are nearly model-independent and contain essential information of the full WMAP CMB and SDSS BAO data [20]. The shift parameter R is given by [20, 21]

$$R \equiv \Omega_{\rm m0}^{1/2} \int_0^{z_*} \frac{dz}{E(z)}, \tag{15}$$

where the redshift of recombination $z_* = 1091.3$, from the WMAP 7-year data [11]. The shift parameter R relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at z_* and the angular scale of the first acoustic peak in the CMB power spectrum of temperature fluctuations [20, 21]. The value of R is 1.725 ± 0.018, from the WMAP 7-year data [11]. On the other hand, the distance parameter A from the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [22] is given by

$$A \equiv \Omega_{\rm m0}^{1/2} E(z_{\rm b})^{-1/3} \left[\frac{1}{z_{\rm b}} \int_0^{z_{\rm b}} \frac{dz}{E(z)}\right]^{2/3}, \tag{16}$$

where $z_{\rm b} = 0.35$. In Ref. [23], the value of A has been determined to be $0.469\ (n_{\rm s}/0.98)^{-0.35} \pm 0.017$. Here, the scalar spectral index $n_{\rm s}$ is taken to be 0.963, from the WMAP 7-year data [11]. So the total χ^2 is given by

$$\chi^2 = \tilde{\chi}^2_{\rm SN} + \chi^2_{\rm CMB} + \chi^2_{\rm BAO}, \tag{17}$$

where $\tilde{\chi}^2_{\rm SN}$ is given by (14), $\chi^2_{\rm CMB} = (R - R_{\rm obs})^2/\sigma_R^2$ and $\chi^2_{\rm BAO} = (A - A_{\rm obs})^2/\sigma_A^2$. The best-fitted model parameters are determined by minimizing the total χ^2.

Now we fit the RDE and IRDE models to the observational data. We use the MCMC method and finally we obtain the best-fit parameters and the corresponding $\chi^2_{\rm min}$. The best-fit, 1σ and 2σ values of the parameters with $\chi^2_{\rm min}$ of the four models are all presented in Table I. For convenience, in the table we abbreviate the three interacting models: IRDE1 corresponds to $Q = 3bH\rho_{\rm de}$, IRDE2 corresponds to $Q = 3bH\rho_{\rm m}$, and IRDE3 corresponds to $Q = 3bH(\rho_{\rm de} + \rho_{\rm m})$.

TABLE I: The fit results of the ΛCDM, RDE and IRDE models. Here, IRDE1 model corresponds to $Q = 3bH\rho_{\rm de}$, IRDE2 corresponds to $Q = 3bH\rho_{\rm m}$, and IRDE3 model corresponds to $Q = 3bH(\rho_{\rm de} + \rho_{\rm m})$.

Model	$\Omega_{\rm m0}$	b	α	$\chi^2_{\rm min}$	ΔAIC	ΔBIC
ΛCDM	$0.270^{+0.014+0.028}_{-0.013-0.026}$	N/A	N/A	542.919	0	0
RDE	$0.323^{+0.022+0.037}_{-0.021-0.035}$	N/A	$0.356^{+0.021+0.035}_{-0.021-0.035}$	565.683	24.764	29.090
IRDE1	$0.273^{+0.034+0.051}_{-0.032-0.048}$	$0.052^{+0.011+0.017}_{-0.015-0.025}$	$0.442^{+0.045+0.069}_{-0.044-0.066}$	542.698	3.779	12.431
IRDE2	$0.269^{+0.034+0.052}_{-0.032-0.049}$	$0.032^{+0.013+0.021}_{-0.013-0.019}$	$0.433^{+0.043+0.066}_{-0.042-0.062}$	543.038	4.119	12.771
IRDE3	$0.270^{+0.034+0.052}_{-0.032-0.048}$	$0.020^{+0.006+0.010}_{-0.007-0.011}$	$0.437^{+0.045+0.068}_{-0.043-0.064}$	542.875	3.956	12.608

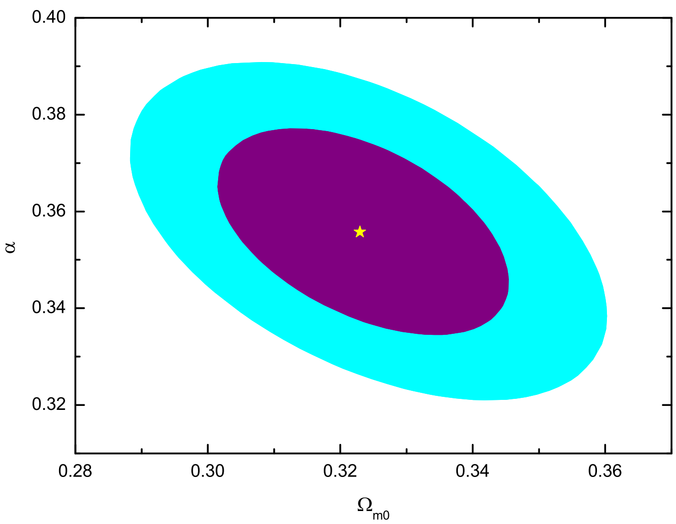

FIG. 1: The probability contours at 1σ and 2σ confidence levels in the $\Omega_{\rm m0} - \alpha$ plane for the RDE model.

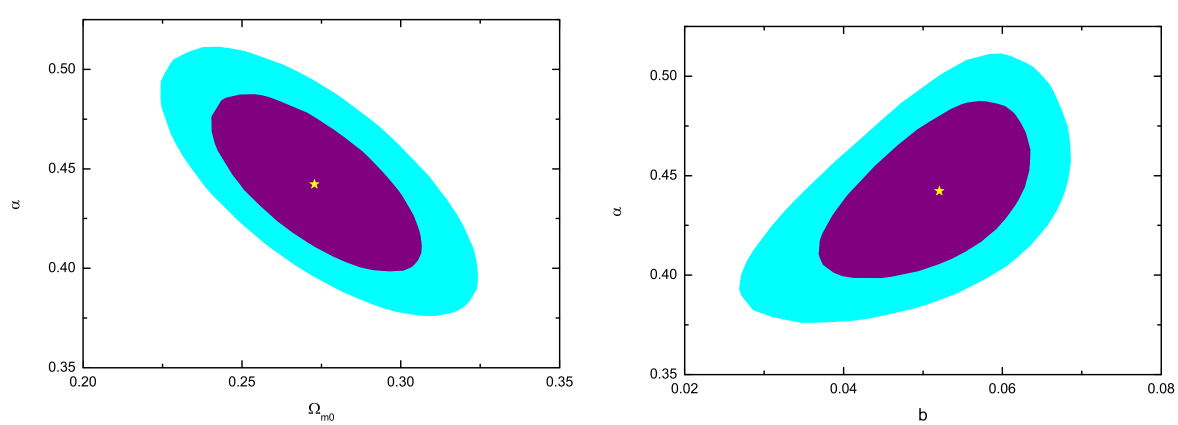

FIG. 2: The probability contours at 1σ and 2σ confidence levels in the $\Omega_{\rm m0} - \alpha$ and $b - \alpha$ planes for the IRDE1 model (corresponding to $Q = 3bH\rho_{\rm de}$).

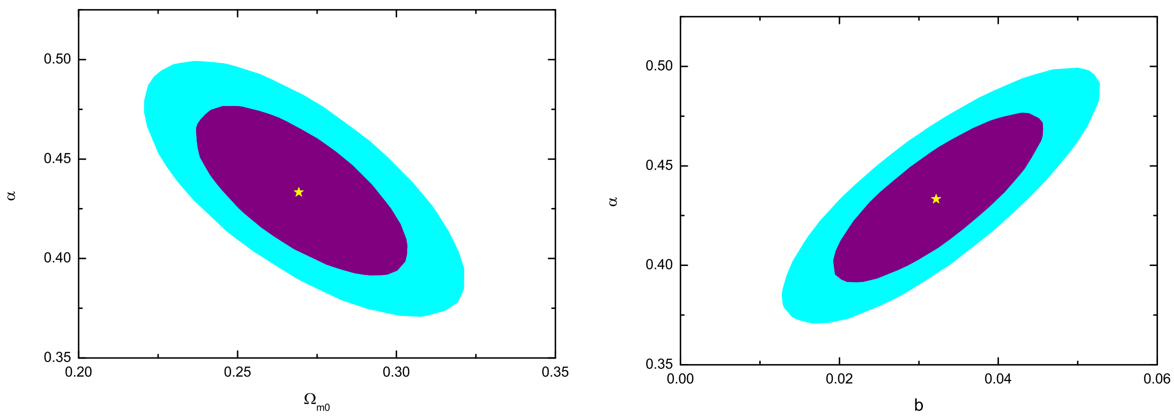

FIG. 3: The probability contours at 1σ and 2σ confidence levels in the $\Omega_{\rm m0}-\alpha$ and $b-\alpha$ planes for the IRDE2 model (corresponding to $Q=3bH\rho_{\rm m}$).

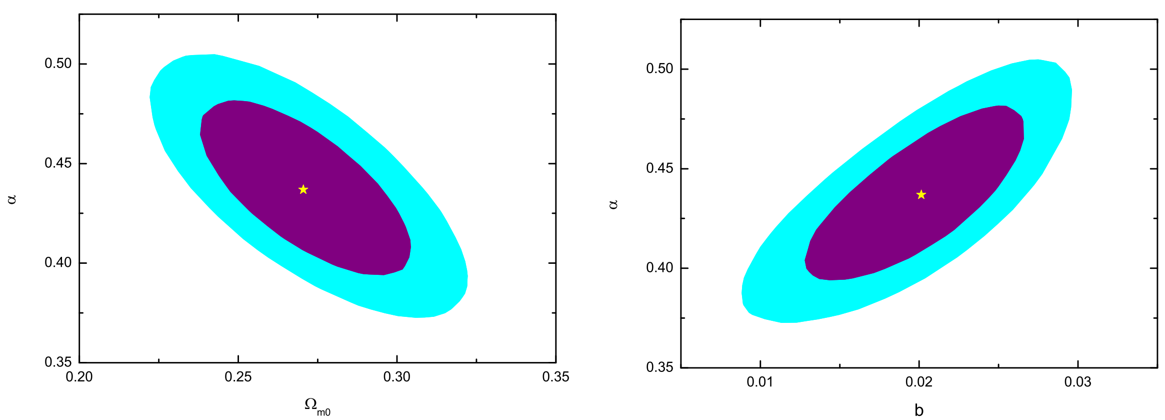

FIG. 4: The probability contours at 1σ and 2σ confidence levels in the $\Omega_{\rm m0}-\alpha$ and $b-\alpha$ planes for the IRDE3 model (corresponding to $Q=3bH(\rho_{\rm de}+\rho_{\rm m})$).

For the RDE model, we obtain the best-fit values of the parameters: $\Omega_{\rm m0}=0.323$ and $\alpha=0.356$, and the corresponding minimal χ^2 is $\chi^2_{\rm min}=565.683$. Obviously, the fit value of $\Omega_{\rm m0}$ in the RDE model is rather big, evidently greater than that of the Lambda Cold Dark Matter (ΛCDM) model, 0.270. Note that for making a comparison with the ΛCDM model, we also fit the ΛCDM model and list the results also in Table I. Now, let us see the fit results of the IRDE models. We find that, for all the three IRDE models, the best-fit values obtained for $\Omega_{\rm m0}$ and α are approximately equal: $\Omega_{\rm m0}=0.27$ and $\alpha=0.44$ (for IRDE2, $\alpha=0.43$). So, it is clearly seen that the fit value of $\Omega_{\rm m0}$ in the IRDE models is more reasonable, consistent with that of the ΛCDM model. Comparing the values of α, it is found that, when there is an interaction between dark energy and cold dark matter, the value of α tends to become bigger. From the above analysis on the fitting results, we find that the parameters $\Omega_{\rm m0}$ and α should be in anti-correlation. For the coupling constant b in the IRDE models, we see clearly that the values of b are explicitly different for the three IRDE models. The value of b in the IRDE1 model is the biggest, $b=0.052$; the value of b in the IRDE3 model is the smallest: $b=0.020$. Since in the IRDE models there is an additional parameter, b, the values of $\chi^2_{\rm min}$ should be substantially less than that of the RDE model. This is the case: the

three IRDE models have the similar values of $\chi^2_{\rm min}$, all around 543, much smaller than that of the RDE model. Among the three IRDE model, the IRDE1 model has the smallest $\chi^2_{\rm min}$, 542.698.

However, it is unwise to use the χ^2 statistic solely to compare competing models since the number of parameters is different for the models. In general, a model with more parameters tends to give a lower $\chi^2_{\rm min}$, and so one may employ the information criteria (IC) to assess the worth of models. These statistics favor models that give a good fit with fewer parameters. Thus, we shall use the AIC (Akaike information criterion) and BIC (Bayesian information criterion) as model selection criteria, defined, respectively, as

$$\mathrm{AIC} = \chi^2_{\rm min} + 2k, \quad \mathrm{BIC} = \chi^2_{\rm min} + k\ln N, \tag{18}$$

where k is the number of parameters, and N is the data points used in the fit. Note that the absolute value of the criterion is not of interest, only the relative value between different models, ΔAIC or ΔBIC, is useful. For the details about the AIC and BIC, especially their applications in a cosmological context, see, e.g., Refs. [24–28]. We give the calculation results of AIC and BIC for the models in Table I. Since the ΛCDM model is an important reference model for the studies of dark energy models, we also include its results in the table. From Table I, we see that, though the value of $\chi^2_{\rm min}$ of the ΛCDM model is greater than that of the IRDE models, the ΛCDM model gives the lowest values of AIC and BIC. This result is consistent with the previous studies [26–28]. Actually, the ΛCDM model is the best model fitting the current data among all of the existing dark energy models. Therefore, the ΛCDM model is chosen to be a fiducial model when we present the values of ΔAIC and ΔBIC. Obviously, the RDE model is strongly unsupported by the data, since its ΔBIC is fairly high; see also Ref. [27] for comparison of various dark energy models including the RDE model. Comparing with the RDE model, the IRDE models are much better, though they are still worse than the ΛCDM model.

Figures 1−4 show the probability contours at 68.3% and 95.4% confidence levels in $\Omega_{\rm m0}-\alpha$ and $b-\alpha$ parameter planes for the RDE and IRDE models. We find that the $\Omega_{\rm m0}-\alpha$ planes of the IRDE models show a stronger degeneracy than that of the RDE model. In addition, for all the three IRDE models, we see that $\Omega_{\rm m0}$ and α are anti-correlated, and b and α are in positive correlation.

It should also be noted that, in the work of Suwa and Nihei [17], the contour plots in the parameter planes are incomplete: when plotting the $\alpha-\Omega_{\rm de0}$ contours, $\gamma = 3b$ is fixed to be 0.15; when plotting the $\gamma-\alpha$ contours, $\Omega_{\rm de0}$ is fixed to be 0.73; see Figs. 2 and 3 in Ref. [17]. In our work, the probe of the parameter-space is complete, i.e., when plotting the contours in parameter plane, other nuisance parameters are marginalized. In addition, it should be mentioned that the IRDE1 case was also investigated in Ref. [28]. In Ref. [28], the authors analyzed in detail the various holographic dark energy models, with the IR cutoff chosen to be the Hubble scale, the future event horizon, and the Ricci scale, respectively, with an interaction between dark energy and dark matter. For the IRDE1 model, they also constrain the model by using the current data, but they assume $w_0 = -1$ in their analysis. The result of the coupling constant they obtained is $b = 0.05 \pm 0.01$, almost the same as ours. Therefore, we find that our study confirms the results of Ref. [28]. For other relevant work, see also, e.g., Ref. [29].

In summary, we have extended the RDE model to include the possible interaction between dark energy and cold dark matter in this paper. Though in Refs. [17, 28] a specific form of the interaction in the RDE model has been investigated, our study makes some significant improvements. In this work, we have considered three phenomenological forms for Q in the RDE model, namely, $Q = 3bH\rho$, where ρ can be taken to be $\rho_{\rm de}$, $\rho_{\rm m}$ and $\rho_{\rm de}+\rho_{\rm m}$, respectively. We have solved the IRDE models, and got a uniform analytical solution for the three IRDE models. Furthermore, we constrained the RDE and IRDE models by using the latest observational data, including the 557 Union2 SN data, the CMB WMAP 7-yr data, and the BAO SDSS data. Our fit results show that

when the interaction between dark energy and dark matter is considered in the RDE model, a more reasonable value of Ω_{m0} will be obtained, i.e., $\Omega_{m0} = 0.27$, rather than the value of $\Omega_{m0} = 0.32$ in the RDE model without interaction. Moreover, it has been shown that a rather strong coupling between dark energy and cold dark matter in the RDE model is favored by the observations: the best-fit value of b is around $\mathcal{O}(10^{-2})$, in particular, for the IRDE1 model, $b = 0.052$. Our work makes the study of the interacting model of the holographic Ricci dark energy more complete.

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